Short Tricks - JEE-Main

By

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1. Progression

Lecture - 2

Q.8 The rational number which equals the number $2.\overline{357}$ with recurring decimal is -

(A) $\frac{2355}{1001}$ (B) $\frac{2379}{997}$ (C) $\frac{2355}{999}$ (D) None of these

Sol. **[C]**

Proper Method	Short Trick
$2.\overline{357} = 2 + 0.357 + 0.000357 + \ldots\ \infty$ $= 2 + \frac{357}{10^3} + \frac{357}{10^6} + \ldots\ \infty$ $= 2 + \frac{\frac{357}{10^3}}{1-\frac{1}{10^3}} = 2 + \frac{357}{999} = \frac{2355}{999}$	$2.\overline{357} = \frac{2357-2}{999}$ $= \frac{2355}{999}$

Q.9 If S_n denotes the sum of n terms of an A.P., then $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ is equal to -

(A) 0 (B) 1 (C) 1/2 (D) 2

Sol. **[A]**

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Proper Method

In AP : S_n

$\boxed{T_1, T_2, T_3 \ldots\ldots\ldots, T_n}$ $T_{n+1}, T_{n+2}, T_{n+3}$

Clearly $S_{n+3} = S_n + T_{n+1} + T_{n+2} + T_{n+3}$

$S_{n+2} = S_n + T_{n+1} + T_{n+2}$

$S_{n+1} = S_n + T_{n+1}$

Putting in

$S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$

We get $= T_{n+1} - 2T_{n+2} + T_{n+3}$

$= (T_{n+1} + T_{n+3}) - 2(T_{n+2}) = 0$

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Short Trick

Let the A.P. is

1, 2, 3, 4, 5, 6, ...

$S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$

Put n = 1

$\Rightarrow S_4 - 3S_3 + 3S_2 - S_1$

$= 10 - 3(6) + 3(3) - 1 = 0$

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Q.10 Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals –

(A) 0

(B) 1

(C) 1/mn

(D) $\frac{1}{m} + \frac{1}{n}$

Sol. [A]

Proper Method	Short Trick
$T_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$ ......(i) & $T_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$ ......(ii) (i) – (ii) $(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$ $\therefore \boxed{d = \frac{1}{mn}}$ From (i) $a + \frac{(m-1)}{mn} = \frac{1}{n}$ $\Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$ $\therefore \boxed{a = \frac{1}{mn}}$ $\therefore a - d = 0$	Let $m = 1$, $n = 2$, then $T_1 = \frac{1}{2}$ and $T_2 = 1$ $\Rightarrow d = \frac{1}{2}$ $\Rightarrow a - d = 0$

Q.11 If a^2, b^2, c^2 are in A.P. then $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+d}$ are in-

(A) A.P. (B) G.P. (C) H.P. (D) None of these

Sol. [A]

Proper Method	**Short Trick**
$\because$ a^2, b^2, c^2 are in A.P. $\therefore$ $a^2 + ab + bc + ca$, $b^2 + bc + ca + ab$, $c^2 + ca + ab + bc$ ..... are also in A.P. [adding $ab + bc + ca$] or $(a + c)\cdot(a + b)$, $(b + c)(a + b)$, $(c + a)(b + c)$ ... are also in A.P. or $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+d}$ are in A.P. [dividing by $(a + b)(b + c)(c + a)$]	Let $a^2 = 1$, $b^2 = 25$ and $c^2 = 49$ Now $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ $\equiv \frac{1}{12}, \frac{1}{8}, \frac{1}{6}$ Which are in A.P.

Q.12 If $a_1, a_2, a_3.....a_n$ are in AP where $a_i > 0$ for all i then the value of

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+...+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=$$

(A) $\frac{1}{\sqrt{a_1}+\sqrt{a_n}}$ (B) $\frac{1}{\sqrt{a_1}-\sqrt{a_n}}$ (C) $\frac{n}{\sqrt{a_1}-\sqrt{a_n}}$ (D) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

Sol. **[D]**

Proper Method	Short Trick
Let d be the c.d. of the A.P. Now L.H.S. $= \frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2}+\frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3}+\ldots\frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n}$ (Note) $=-\left(\frac{\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}-\sqrt{a_3}+\ldots\sqrt{a_n}}{d}\right)$ $=-\frac{(\sqrt{a_1}-\sqrt{a_n})}{d}=\frac{1}{d}\frac{(a_n-a_1)}{\sqrt{a_n}+\sqrt{a_1}}$ $=\frac{(n-1)d}{d\left[\sqrt{a_n}+\sqrt{a_1}\right]}$ $[\because\ a_n=a_1+(n-1)d]$ $=\frac{n-1}{\sqrt{a_n}+\sqrt{a_1}}$	**Short Trick** Let $n=3$ and $a_1=1$, $a_2=25$, $a_3=49$ then $\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}$ $=\frac{1}{6}+\frac{1}{12}=\frac{1}{4}$ by option (D) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ $=\frac{3-1}{1+7}=\frac{1}{4}$ is the correct answer

Q.13 The sum to n terms of the series $1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + \ldots$ is given by-

(A) n^2 (B) $n(n + 1)$ (C) $n(1+1/n)^2$ (D) None of these

Sol. [A]

Proper Method	Short Trick
Let S be the sum of n terms of the given series and $x = 1 + 1/n$, Then, $S = 1 + 2x + 3x^2 + 4x^3 + \ldots. + n\,x^{n-1}$ $\Rightarrow xS = x + 2x^2 + 3x^3 + \ldots. + (n-1)\,x^{n-1} + nx^n$ $\therefore S - xS = 1 + [x + x^2 + \ldots + x^{n-1}] - nx^n$ $\Rightarrow S\,(1-x) = \dfrac{1-x^n}{1-x} - n\,x^n$ $\Rightarrow S\,(-1/n) = -n[1-(1+1/n)^n] - n\,(1+1/n)^n$ $\Rightarrow \dfrac{1}{n}.\,S = n\,[1-(1+1/n)^n + (1+1/n)^n] = n$ $\Rightarrow S = n^2$	Put $n = 2$ then sum of two terms $= 1+2\left(1+\dfrac{1}{2}\right) = 4$ by option (A) n^2 is correct answer

Q.14 If p^{th}, q^{th} and r^{th} terms of H.P. are u, v, w respectively, then the value of the expression $(q - r)vw + (r - p)wu + (p - q)uv$ is-

(A) 1 (B) 0 (C) – 2 (D) – 1

Sol. [B]

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Proper Method

Let H.P. be

$$\frac{1}{a}+\frac{1}{a+d}+\frac{1}{a+2d}+\ldots\ldots$$

$$u=\frac{1}{a+(p-1)d},\ v=\frac{1}{a+(q-1)d},$$

$$w=\frac{1}{a+(r-1)d}$$

$$a+(p-1)d=\frac{1}{u},\ a+(q-1)\,d=\frac{1}{v},$$

$$a+(r-1)d=\frac{1}{w}$$

$\Rightarrow (q-r)\,\{a+(p-1)d\}+(r-p)\,\{a+(q-1)\,d\}+\ldots\ldots$

$=\frac{1}{u}\,(q-r)+\frac{1}{v}\,(r-p)+\ldots.$

$\Rightarrow\ (q-r)\,vw+\ldots..=0$

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Short Trick

Let $p=1,\ q=2,\ r=3$

and $u=2,\ v=3,\ w=6$

$(q-r)vw+(r-p)wu+(p-q)uv$

$=-18+24-6=0$

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